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Fracture Mechanics and the Design of Wood Structures [and Discussion]

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Fracture mechanics and the design of wood structures

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[Plate 1]

Applications of linear-elastic fracture mechanics concepts for treating brittle fracture of wood and wood products are discussed. The influence of orthotropic elasticity and heterogeneity of wood structure on the computational and experimental aspects of wood fracture are introduced. Several applications of fracture mechanics for the development of design criteria in timber engineering are discussed to illustrate the potential benefits of application of fracture mechanics in design codes.

Introduction

Experience with the structural performance of wood structures designed in accordance with linear-elastic fracture mechanics (l.e.f.m.) criteria is extremely limited since, at present, only the S.A.A. Australian Timber Engineering Code AS 1720–1975 incorporates l.e.f.m. concepts in design (S.A.A. 1975). However, in recent years considerable interest in the application of l.e.f.m. has developed. Many applications have been identified in which the reliability of wood structural elements is improved through application of the more refined l.e.f.m. design criteria. Undoubtedly, these advances will result in the introduction of l.e.f.m. concepts into more timber engineering codes during this decade.

One of the major factors contributing to the use of fracture mechanics concepts originates from the need to relate test results from small laboratory test specimens to the behaviour of wood in structures. Until recently, design stresses and design criteria presented in timber design codes were almost entirely based on results derived from tests on small, clear specimens. For example, design stresses for products such as dimension lumber would be obtained by modifying the strength values obtained from a small, clear test specimen to account for effects of 'defects', such as knots, cracks and holes, which were expected in the commercial product. Recent emphasis on full-size specimen assessment of properties in products such as lumber has indicated that inconsistencies exist in the development of design stresses and that they can be attributed, in part, to a lack of adequate understanding of the effects of defects on strength and also of the importance of size on fracture strength.

In wood, as with most other materials, large members tend to have lower strengths than small members with similar geometry and loading conditions. Differences in strength are generally attributable to the effects of defects that tend to become more numerous and larger as member size increases. In wood, the 'defects' may be natural characteristics such as knots or knot holes, or natural fissures such as resin pockets, which are typically present in trees; or they may be defects introduced during manufacturing processes or construction operations. Defects falling into this last category include seasoning or drying checks, interflake voids in particle and flake boards, butt joints between the veneers (or lamina) in glued products such as plywood, or sharp-cornered details such as notches that may be design features or accidentally introduced into structural components.

Through application of l.e.f.m. concepts, it has been possible to develop new design criteria that recognize, in a direct way, the relation between defect type, defect size, member geometry and ultimate capacity. Emphasis on large-scale member behaviour has also focused attention on brittle fracture problems that can be treated by using concepts of statistical size effects, such as Weibull's weakest-link models.

The objective in this paper is to review some of the methodology available for treating brittle fracture problems in wood products containing crack-like defects, while demonstrating some of the refinements of the l.e.f.m. concept that must be recognized to treat wood fracture problems. The discussion begins with a brief examination of the influence of wood anistropy on the calculation of stress intensity factors and on the subsequent measurement of the corresponding critical parameters. This discussion will be followed by a series of specific examples illustrating the application of the l.e.f.m. concept to wood engineering problems.

Effect of material anistropy and inhomogeneity

Wood is a highly anisotropic material in both its elastic and strength properties. Its structure is somewhat similar to a linearly reinforced composite material with filament and matrix components, in which elongated wood fibres could be compared to the filament reinforcement and the amorphous intercellular material to the matrix component. An element of typical softwood structure is shown in figure 1. At sufficient distances from the growth centre of the tree stem, the curvature of the annual growth rings can be neglected and, in such cases, three orthogonal planes of material symmetry are identified in relation to the radial, longitudinal and tangential axes shown in figure 1. Owing to the highly orientated nature of the wood structure, both strength and stiffness are great in the longitudinal direction and relatively low in the tangential and radial directions, being approximately 10% of the corresponding longitudinal property, depending on species. The large degree of structural anisotropy contributes to the formation of natural cleavage planes in which cracks tend to form and propagate. For wood, the longitudinal–radial and longitudinal–tangential planes are natural cleavage plans, and crack propagation therefore typically occurs along the grain. In fact, it is extremely difficult to produce crack propagation across the grain (i.e. normal to the longitudinal direction).

Owing to the material's anisotropy, it is necessary to identify the system of crack propagation for each of the three modes of crack-tip deformation. For each mode of propagation (mode I, mode II and mode III), six principal systems of propagation exist, as shown in figure 2. A system of propagation is identified with a pair of letters, the first indicating the direction normal to the crack surface and the second indicating the direction of propagation in the crack plane. Crack growth across the grain in the l.t. and l.r. systems deserves special consideration, since collinear propagation from a prexisting crack normal to the longitudinal direction does not occur.

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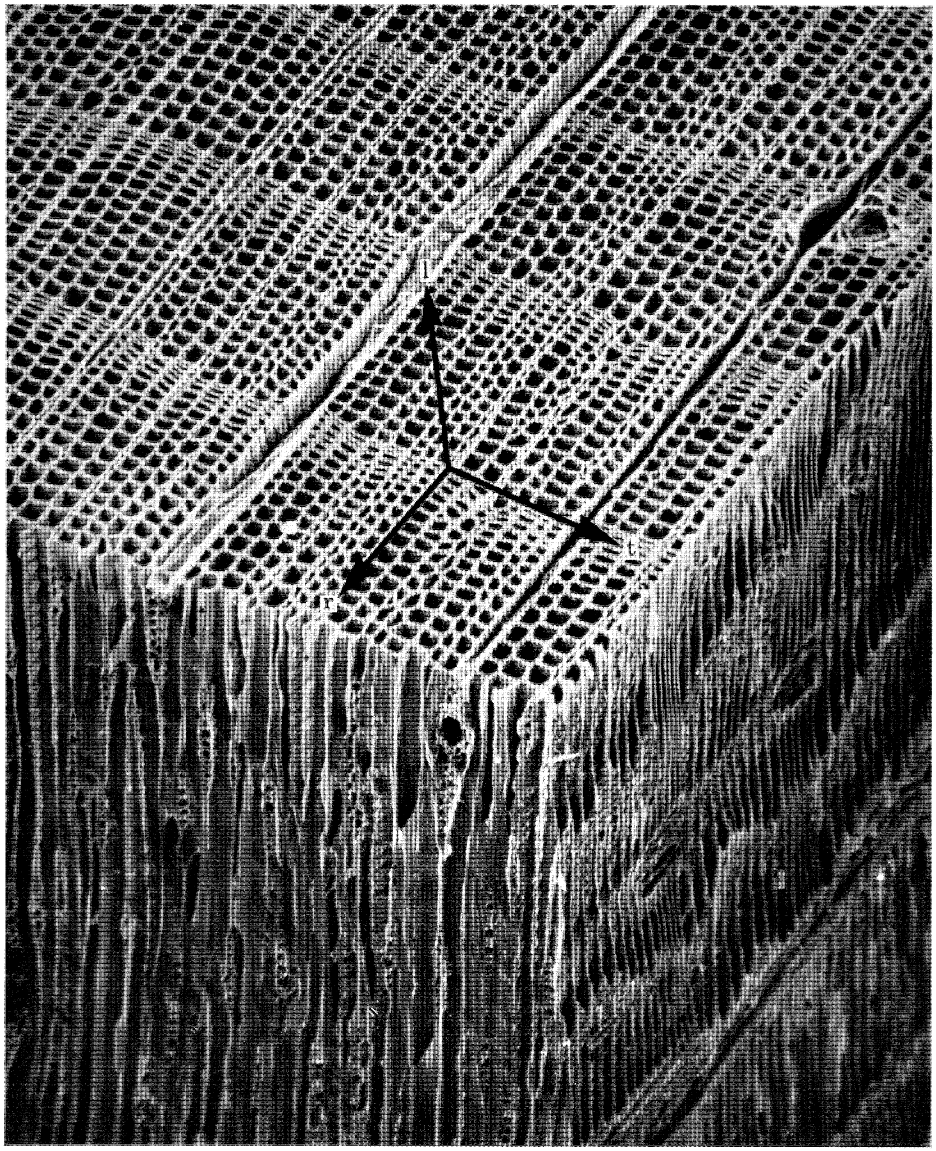

FIGURE 1. Typical softwood structure showing orientation of longitudinal (l), radial (r) and tangential (t) directions.

(*Facing p.* 218)

Computed stress intensity factors

The state of plane stress, σ_{ij}, at the tip of a zero-angle sharp crack (figure 3) is given by

$$\sigma_{ij} = K f_{\mathrm{I}ij}(\theta)/(2\pi r)^{0.5} + K_{\mathrm{II}}\, g_{ij}(\theta)/(2\pi r)^{0.5}, \tag{1}$$

where K_{I} and K_{II} are stress intensity factors; r, θ are the polar coordinates; and f_{ij}, g_{ij} are functions of the angular coordinate θ. For an infinite plate with a crack of length $2a$ in the stress field of figure 3:

$$\left.\begin{aligned} K_{\mathrm{I}} &= \tilde{\sigma}_y(\pi a)^{0.5} H_1, \\ K_{\mathrm{II}} &= \tilde{\tau}_{xy}(\pi a)^{0.5} H_2, \end{aligned}\right\} \tag{2}$$

where H_1 and H_2 are the finite geometry correction factors; $H_1 = H_2 = 1$ for an infinite plate.

Thus, the level of stress, σ_{ij}, is controlled by the magnitude of the stress intensity factors K_{I} and K_{II}. The form of (2) remains constant for all sharp-crack applications, independent of material and member geometry. Values of H are tabulated for many applications and H has

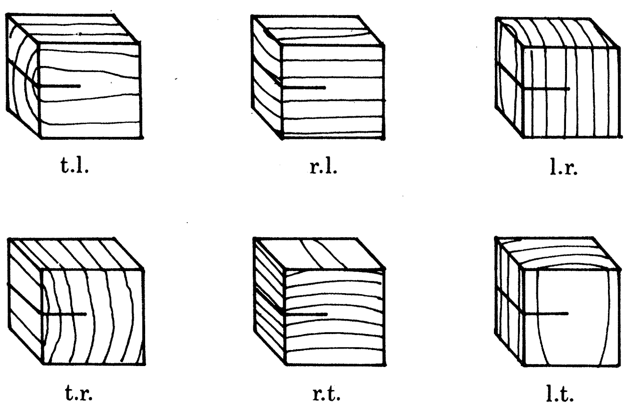

Figure 2. Schematic view of wood blocks showing six principal systems of crack propagation. First letter indicates the normal to crack surface and the second letter indicates the direction of crack propagation.

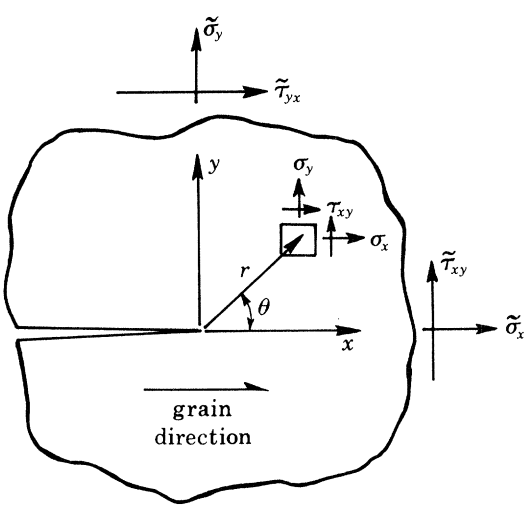

Figure 3. Coordinate system used for definition of singular stress field at tip of sharp cracks. Global stresses are differentiated from local crack tip stresses by tilde symbol.

been shown to be independent of material properties for isotropic materials. Walsh (1972) investigated the effect of orthotropy on computed stress intensity factors for several specimen configurations, including double cantilever beams, centre and edge notched plates, and double edge notched plates. For rectangular specimens of sufficient length, orthotropic and isotropic results agree closely; however, as the specimen approaches a square configuration, large differences between orthotropic and isotropic results can occur, with differences also dependent upon grain orientation (Walsh 1972). For the double cantilever beam specimen, large differences between isotropic and orthotropic values exist for all specimen configurations (Walsh 1972). Thus, to apply the fracture mechanics approach to wood products, care must be taken to determine the dependence of the stress intensity on material properties for the particular specimen geometry. Fortunately for many wood applications of wood, the variation of stress intensity factors with elastic constants appears to be sufficiently small for it to be ignored.

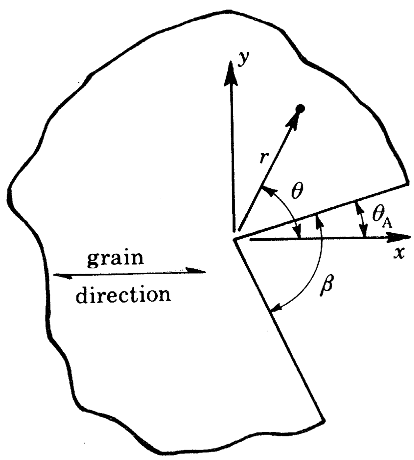

FIGURE 4. Coordinate system used for definitions of singular stress fields at the tip of a sharp notch.

Considerable attention has been given to the analysis of the stress distribution in orthotropic materials at the tip of sharp notches. Leicester (1971) developed a method for the analysis of the stress field at the root of a mathematically sharp notch of arbitrary notch angle β, as shown in figure 4. In general, two singularities of stress exist and the state of stress in the region of a sharp notch is given by

$$\sigma_{ij} = K_A f_{ij}^1(\theta)/(2\pi r)^s + K_B g_{ji}^1(\theta)/(2\pi r)^q \tag{3}$$

(Leicester 1971), where r and θ are the polar coordinates, s and q are constants for a particular notch angle ($0.5 \geqslant s \geqslant q \geqslant 0$) and f_{ij}^1, g_{ij}^1 are constants that are functions of θ. The parameters s and q are functions of the material properties, while the 'stress intensity factors' K_A and K_B depend on the material properties, notch angle, member geometry and loading condition.

Solutions for non-zero angle notches revert to the sharp zero-angle crack solutions of (1) as the notch angle β (figure 4) approaches zero, and under these conditions K_A and K_B correspond to the stress intensity factors K_I and K_{II}. Leicester (1971) defines a size coefficient s, which relates the fracture strength σ of structural materials to a scale parameter L according to

$$\sigma_1/\sigma_2 = (L_2/L_1)^s. \tag{4}$$

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

For sharp cracks ($\beta = 0.0$), the size coefficient $s = 0.5$, as derived for isotropic and orthotropic materials previously. For other notch angles, the size coefficient s varies between $s = 0$ for $\beta = 180°$, and $s = 0.5$ for $\beta = 0$. Tests of the strength of beams show good agreement with theoretical predictions (Leicester 1973).

Stress intensity factors K_A and K_B for isotropic and orthotropic properties have been determined with the use of a calibrated finite element procedure (Walsh 1974). The results suggest that stress intensity factors K_A and K_B differ significantly for isotropic and orthotropic materials, whereas for $\beta = 0$ (a sharp crack) the differences were relatively unimportant.

TABLE 1. MODE I CRITICAL STRESS INTENSITY FACTORS FOR WOOD

species	system of propagation	fracture toughness†, $K_{Ic}/(\mathrm{kN\ m^{-\frac{3}{2}}})$	source
softwoods			
Douglas-fir	t.l.	309	Schniewind & Centeno (1971)
	r.l.	410	
	t.r.	355	
	r.t.	355	
	l.t.	2417	
	l.r.	2692	
western white pine	t.l.	190	Johnson (1973)
western red cedar	t.l.	185	Johnson (1973)
hoop pine	t.l.	494	Walsh (1971)
hardwoods			
hard maple	t.l.	492	Johnson (1973)
paper birch	t.l.	564	
red oak	t.l.	407	
lauan	t.l.	478	
messymate stringbark	t.l.	505	Walsh (1971)
maiden's gum	t.l.	681	Walsh (1971)
balsa	t.l. and r.l.	112	Wu (1963)

† Moisture content *ca.* 12%; temperature *ca.* 21 °C.

CRITICAL STRESS INTENSITY FACTORS

According to the linear-elastic fracture mechanics model, fracture is assumed to occur when the stress intensity factor K achieves a critical value $K = K_c$, a material property that must be determined experimentally. Critical stress intensity factors for sharp cracks have been determined for mode I and mode II fracture for the important systems of propagation.

Mode I critical stress intensity factors, K_{Ic}, have been determined by using a variety of specimens of the type commonly used for isotropic materials. Schniewind & Centeno (1971) presented K_{Ic} values for the six principal systems of propagation for Douglas-fir wood (table 1). Single edge-notched beam specimens tested in the l.t. and l.r. modes gave K_{Ic} values nearly an order of magnitude greater than the values for the other four systems. It is important to note that for the l.t. and l.r. systems, the crack generally does not propagate straight ahead from the machined notch; instead it turns and propagates along the grain. The mechanism of crack propagation in such cases has not been studied in detail, but the Cook–Gordon (Gordon 1971) weak interface concept can explain this behaviour. Some results for other hardwood and softwood species are also given in table 1.

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Mode II fracture of wood occurs nearly as frequently as mode I fracture and it has therefore received much more attention than might appear warranted by those concerned mainly with fracture in isotropic materials. Two approaches have been developed for determining mode II fracture toughness. Leicester (1974*a*) and Walsh (1972) have found the asymmetrically loaded single edge-notched beam specimen of figure 5 useful for determining K_{IIc} for cracks orientated perpendicularly to the grain. The two notch details were considered: a sawn notch (figure 5*a*) and a butt joint (figure 5*b*). For sawn notches, little dependence of fracture strength on notch root detail was found. Barrett & Foschi (1977*a*) have found the end-notched beam specimen shown in figure 5*c* particularly useful for studying mode II fracture for cracks orientated along the grain. K_{II} values for longitudinal propagation have been obtained for a few softwood species (table 2).

Critical parameters, K_{Ac} and K_{Bc}, have not been developed for many cases. In principle, however, notched-beam specimens can be used to obtain K_A and K_B values, and some specific applications will be considered in the next section.

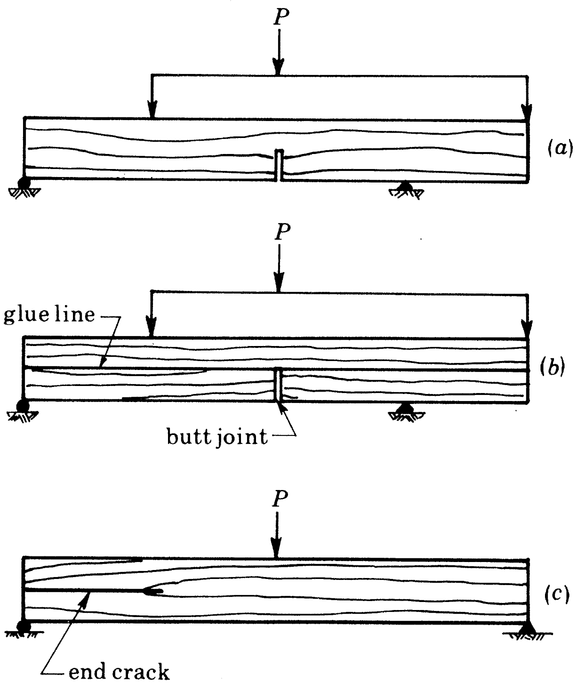

FIGURE 5. Specimen used for determination of K_{IIc}: (*a*) sawn notch; (*b*) butt joint in glued–laminated pieces; (*c*) end-split beam.

TABLE 2. MODE II CRITICAL STRESS INTENSITY FACTORS FOR WOOD

species	system of propagation	fracture toughness†, $K_{IIc}/(\mathrm{kN\ m^{-3/2}})$	source
white spruce	t.l.	1890	J. D. Barrett (unpublished)
lodgepole pine	t.l.	2187	
amabilis fir	t.l.	1626	
Douglas-fir	t.l.	2143	
balsa	t.l.	159	Wu (1963)

† Moisture content *ca.* 12%; temperature *ca.* 21 °C.

Applications

This brief background was presented to illustrate some special aspects of wood fracture that require attention before linear-elastic fracture mechanics can be properly applied in wood and wood structures. Applications to fracture problems of current interest in timber engineering will now be illustrated.

Effect of knots on lumber strength

Currently, design stresses for dimension lumber are related to the strength of small, clear specimens. The small, clear specimen test results are modified for several strength-influencing factors, including knots. Adjustment factors currently applied were developed without the aid of a satisfactory theory of failure for structural lumber. Pearson (1974) presented a fracture mechanics approach for evaluating the effects of knots on the strength of structural lumber by using a single-edge notch and centre-notched plate strips to model edge and centre knots. Tests of southern pine lumber showed that consistent results could be obtained by assuming that a straight crack perpendicular to the grain represented the knot. The appropriate fracture mechanics models were then used to develop relations between knot diameter and member width. These relations can be used to develop grades of structural lumber with desired bending and tensile strength levels based on critical stress intensity factors for the l.t. and l.r. systems of propagation. Kunesh & Johnson (1972) also studied the relation between knot size and tensile strength for boards with single centre and single edge knots. Foschi & Barrett (1980) fitted single-edge notch and centre-notch fracture mechanics models to these data, and comparison of edge and centre knot results shows that for a given knot diameter, edge knots have a much more severe effect on tensile strength than centre knots of the same diameter. The calibrated fracture mechanics model was then used successfully to account for differences in tensile strength of special grades of lumber used to produce glued–laminated beams.

Shear fracture of bending members

In developing design shear stresses for structural lumber it is necessary to consider the effects of end splits (figure 5*c*) on ultimate capacity. These splits, which tend to arise during the drying of lumber, generally occur near the mid-depth of a beam. When these cracks are sufficiently large, the shear capacity of the member can be controlled by the mode II fracture toughness of the material. Barrett & Foschi (1977*b*) developed a fracture criterion that can be used to predict the effect of end splits on shear capacity. For uniformly distributed loads on simply supported beams, the ultimate shear stress, $\tau = 1.5\,V/A$, is given by

$$\tau(p) = \frac{0.13}{\sqrt{d}}\left(\frac{3}{1+2a/d}\right) K_{\mathrm{IIc}}(p), \tag{5}$$

where d is the member depth, a is the crack length, p is the specified failure probability level, V is the maximum shear force and A is the cross-sectional area of the beam.

Thus, the fracture mechanics approach provides the theoretical framework for the development of a rational model of shear capacity that can be expanded to treat splits parallel to the grain at any location for any loading condition.

Notched beams

Notched beams are not commonly used in structures because of the potentially large strength reducing effect of the notch. However, for those cases where notching is required, provisions for designing notched beams have been developed by using l.e.f.m. concepts. For example, the fracture strength of rectangular beams notched on the tension face is treated in the S.A.A. Australian Timber Engineering Code AS 1720–1975 (S.A.A. 1975). A conservative design criterion is presented in which the design capacity is given as a function of the nominal bending stress and the nominal shear stress (figure 6) according to

$$0.3\tau_{\mathrm{nom,\,b}} + 0.7\tau_{\mathrm{nom,\,s}} = C_3 f_{\mathrm{v}}, \tag{6}$$

where $\tau_{\mathrm{nom,\,b}} = 6M/Bd_{\mathrm{min}}^2$, $\tau_{\mathrm{nom,\,s}} = 1.5V/Bd_{\mathrm{min}}$, f_{v} = shear block strength for the species of interest and C_3 is a constant tabulated in table 3 for some specific notch angles.

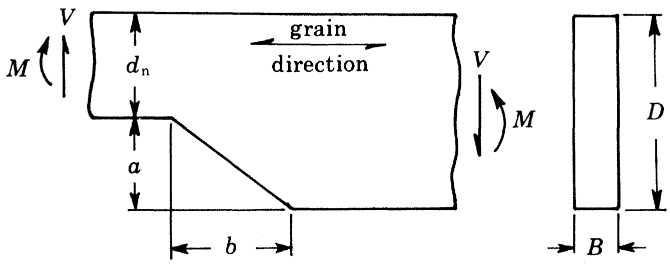

FIGURE 6. Notch beam geometry used in development of failure criterion for notched beams.

TABLE 3. PARAMETER C_3 FOR SELECTED NOTCH ANGLES

notch slope	$a \geqslant 0.1\,D$	$a < 0.1\,D$
$b/a = 0$	$3/D^{\frac{1}{2}}$	$1/a^{\frac{1}{2}}$
$b/a = 2$	$2.6/D^{\frac{1}{3}}$	$1.2/a^{\frac{1}{3}}$
$b/a = 4$	$2.2/D^{\frac{1}{4}}$	$1.3/a^{\frac{1}{4}}$

Butt joints in laminated members

Laminating is a simple and effective process for producing large sections from small elements or lamina. Laminated members can have considerably higher design strengths than the individual lamina from which they are produced, since knots and other defects tend to be more uniformly distributed through the member. In certain types of laminated products, it is common to arrange several small components end to end within an individual lamination without any type of structural end-joint. When individual elements are placed end to end without any adhesive, as shown in figure 7, a butt joint is formed. Butt joints tend to reduce the member bending and tensile strength. The relation between lamination thickness and strength has been studied by Leicester & Bunker (1969), who suggested an l.e.f.m. approach suitable for development of design criteria. Later, Walsh (1973) studied the interaction of multiples of butt joints in laminated material and presented the following recommendations for joint spacing within and between laminations designed to minimize the effect of butt joints on strength.

(*a*) Joints in the same lamination: no restrictions.

(*b*) Joints in the same section: three clear laminations between joints.

(*c*) Joints in adjacent laminations: spacing of six times lamination thickness.

A conservative failure criterion for predicting the effect of a butt joint on strength, for a uniform stress field, was given by Leicester (1974*b*) as

$$0.2\sigma_x + 1.2\sigma_{xy} < f_{\mathrm{v}}/\sqrt{t}, \tag{7}$$

where σ_x and σ_{xy} are defined in figure 7, f_v is shear block strength and t is the lamination thickness. Leicester (1974*b*) also discusses the generalization of this design criterion for other loading conditions and joint locations such as butt joints in the surface lamina.

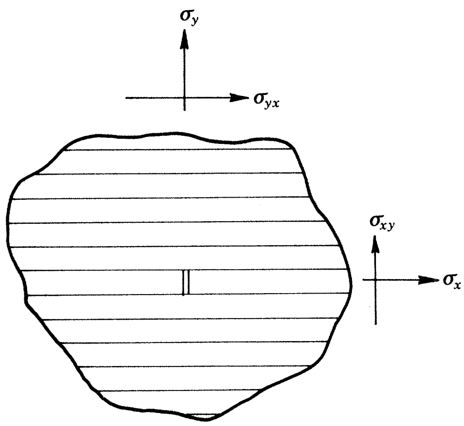

Figure 7. Coordinate system for definition of a uniform stress field at a butt joint in a laminated member.

Delayed fracture and duration of load effects

Wood is a viscoelastic material and is thus subject to creep–rupture effects that require that design stresses for all wood products incorporate a factor to adjust the design stress for the expected period of the design live load. Recently, slow crack growth studies have been utilized to develop a relation between applied stress level and time to failure for wood loaded in tension perpendicular to grain (Mindess *et al.* 1975). Studies indicate that, for cracks growing along the grain, the crack propagation velocity v can be related to the applied stress intensity factor K according to the relation

$$v = AK^n. \tag{8}$$

Material constants A and n have been determined experimentally by using double torsion specimens (Mindess *et al.* 1975) and (9) has been integrated to predict the time to failure for load histories of interest. This type of l.e.f.m. approach offers the potential for development of a fundamental understanding of the failure process controlling delayed fracture in wood products.

Summary

Many opportunities exist for application of linear-elastic fracture mechanics concepts in design of wood structures. Some of these applications have been discussed to illustrate this point but, unfortunately, the engineering design community at present has not been adequately acquainted with the benefits to be achieved through the use of the refined design criteria. Until now, fracture mechanics has essentially been a tool used by a few researchers interested in timber design. Much more work needs to be done to determine the critical stress intensity factors for sharp cracks and notches for the commercially important wood species over the ranges of temperature and moisture content encountered in structural applications. This work must be coupled with additional efforts to demonstrate the range of applicability of the l.e.f.m. concept for wood structures to encourage code authorities to adopt new design philosophies.

I believe that we can expect that shortages of resources, and efforts to reduce costs while maintaining adequate safety, will encourage the continued development and eventual implementation of l.e.f.m. concepts in timber design codes throughout the world.

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Discussion

I. P. Haigh (*Sir Alexander Gibb and Partners, Reading, U.K.*). During the past two days of discussion, the possible application of fracture mechanics to concrete has not been mentioned. This is all the more surprising since Dr Barrett has just described its application in Canada to timber. Discounting the presence in much concrete of steel reinforcement, concrete may still be regarded as an inhomogeneous and composite material, but, except in terms of scale, it appears to be no more inhomogeneous than steel. Has fracture mechanics been applied to concrete?

J. M. Lovegrove (*Department of Civil Engineering, University of Southampton, U.K.*). L.e.f.m. is being used in our work on the fatigue of reinforced concrete beams. We are using the Paris law and the problem is in fact one of fatigue crack growth in an edge-cracked ribbed steel bar of circular cross section. The bar is in tension with a small amount of superimposed bending and a high proportion of the total life is close to the threshold region of the growth rate – K range curve.

In general, when reinforced concrete is loaded sufficiently, the tension concrete cracks and to some extent may debond from the reinforcing bars. This allows localized regions of higher stress to develop in the bars across the cracks in the concrete. It is within these regions that fatigue cracks in the bars can initiate and grow. The bars are of course entirely enclosed within the concrete and the cracks in the concrete are normally about 0.05–0.1 mm wide.

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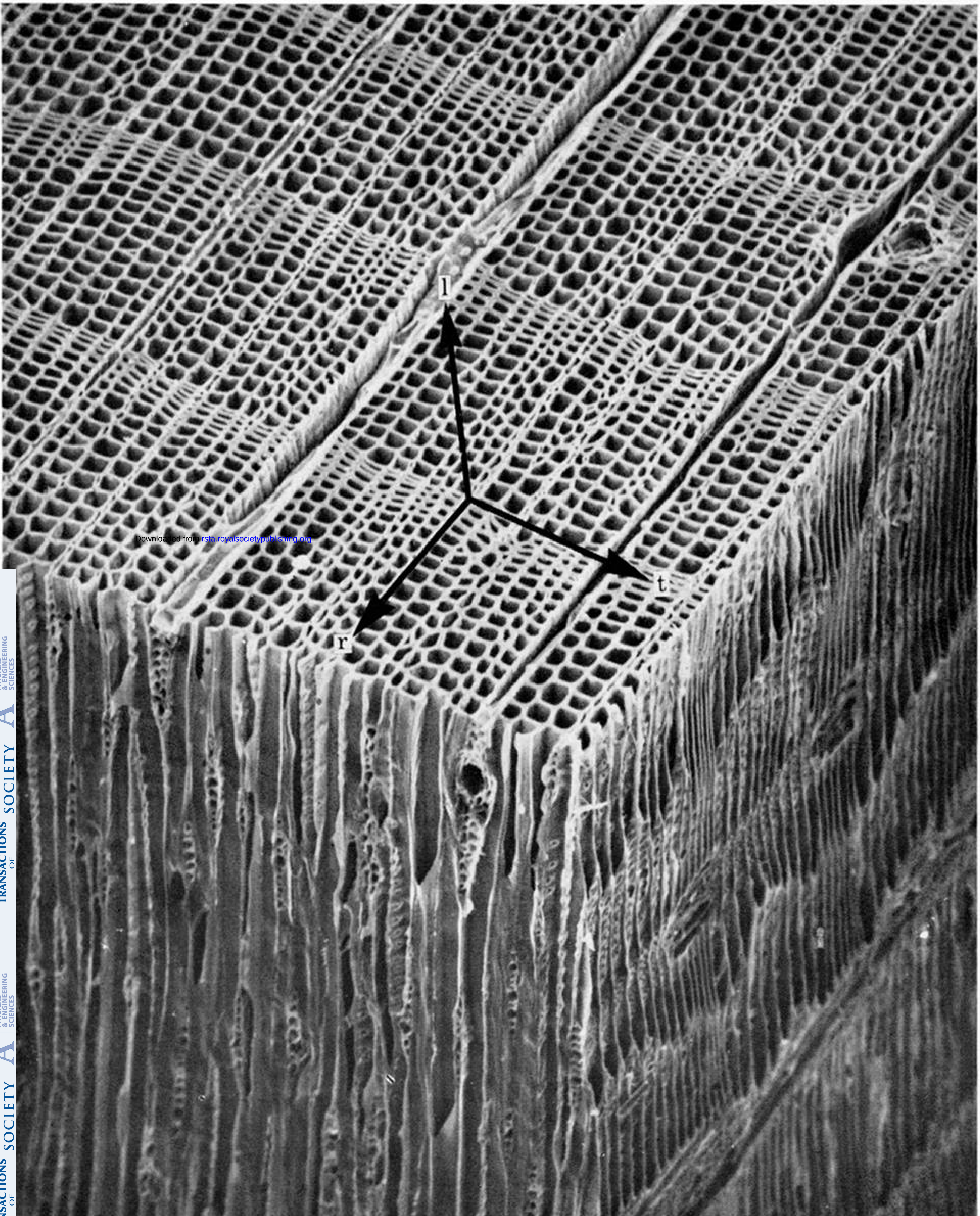

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GURE 1. Typical softwood structure showing orientation of longitudinal (l), radial (r) and tangential (t) directions.